

C3 Numerical Answers**June 2005**

- (b) $\theta_1 = 131.8^\circ; \theta_2 = 228.2^\circ$
- (a) (i) $6 \sin x \cos x + 2 \sec 2x \tan 2x$ (ii) $3(x + \ln 2x)^2(1 + \frac{1}{x})$
- (b) $\frac{2+x}{x}$ (c) $x = \pm 2$
- (a) $3e^x - \frac{1}{2x}$
(c) $x_1 = 0.0613\dots, x_2 = 0.1568\dots, x_3 = 0.1425\dots, x_4 = 0.1445\dots$
- (c) $R = 7.21; \alpha = 0.588$ (d) $\theta = 0, 2.12$
- (c) $a = -2, b = -1$ (d) $x = -\frac{1}{6}$
- (b) 14 years

January 2006

- $\frac{x+3}{x+1}$
- $y = -3x + 9$
- (a) (i) $3x^2e^{3x+2} + 2xe^{3x+2}$ (ii) $\frac{-18x^3 \sin(2x^3) - 3 \cos(2x^3)}{9x^2}$
(b) $\frac{1}{8 \cos\left(\arcsin\left(\frac{x}{4}\right)\right)}$
- (b) $x_1 = 1.41, x_2 = 1.39, x_3 = 1.39$
- (a) $R = \sqrt{160}; \alpha \approx 18.43^\circ$ (b) $x = 38.0^\circ, 285.2^\circ$
(c) (i) $-\sqrt{160}$; (ii) $x \approx 161.57^\circ$
- (c) $\theta = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$
- (c) \mathbb{Z}^+ (d) $x \approx -0.418$

June 2006

- (a) $\frac{3x+2}{x+1}$ (b) $\frac{3x-1}{x}$
- (a) $3e^{3x} + \frac{1}{x}$ (b) $3x(5+x^2)^{\frac{1}{2}}$
- (a) 425°C (b) $t = 7.49$ (c) $(\pm) 1.64^\circ\text{C}/\text{min}$
- (b) $x_1 = 0.2670, x_2 = 0.2809, x_3 = 0.2746, x_4 = 0.2774$
- (c) 135°
- (b) $-\infty < f(x) < \infty$ (c) $\ln\left(\frac{3k}{2}\right)$ (d) $1\frac{1}{2}$
- (a) $-\frac{3\sqrt{7}}{8}$

January 2007

- (b) $\frac{9\sqrt{3}}{16}$
- (c) $y = -\sqrt{2x+2} + \frac{\pi}{4}$
- (i) $(3, \frac{1}{6}), (-3, -\frac{1}{6})$ (ii) 18
- (a) $y = 2 \sin\left(x + \frac{\pi}{3}\right)$ (b) $x = \frac{\pi}{2}, \frac{11\pi}{6}$
- (a) \mathbb{R} (b) $f^{-1}(x) < 2$ and $f^{-1}(x) \in \mathbb{R}$ (c) 1.5, $\ln 4$
(d) $x_1 = -0.3704, x_2 = -0.3452$ (e) $k = -0.352$
- (b) (1, -11) (c) $a = 2, b = 4, c = 4$
- (b) (ii) $\arcsin x = \frac{\pi}{2} - y$ (ii) $\frac{\pi}{2}$

June 2007

- (a) $x = 2$ (b) $x = \ln 3, x = 0$
- (b) $\frac{8}{(2x-1)^2}$
- (a) $\frac{dy}{dx} = x^2 e^x + 2xe^x$ (b) $x = 0, y = 0$ and $x = -2, y = 4e^{-2}$
(c) $\frac{d^2y}{dx^2} = x^2 e^x + 2xe^x + 2xe^x + 2e^x$
(d) $x = 0$ is a min, $x = -2$ is a max
- (b) $x_2 = 0.6455, x_3 = 0.6517, x_4 = 0.6526$
- (a) $\ln 3$ (b) $f^{-1}(x) = \frac{1}{2}(e^x + 1), x \in \mathbb{R}$ (d) $x = 3\frac{2}{3}, x = 2\frac{1}{3}$
- (a) $\sqrt{13} \sin(x + 0.588)$ (b) 169 (c) $x = 2.273$ or $x = 5.976$
- (c) $\theta = 20.9^\circ, 69.1^\circ, 200.9^\circ, 249.1^\circ$
- (a) 5.353 (c) $T = 13.06\dots$

January 2008

- $a = 2, b = 0, c = -1, d = 1, e = 0$
- (b) $y = x$
- (b) $x_1 = 2.50408, x_2 = 2.50498, x_3 = 2.50518$
- (a) (5, 4), (-5, 4) (b) (5, 4), (-5, 4) (c) (4, 8), (-6, -8)
- (a) 1000 (b) 0.000121 (c) 62.5
- (a) $4 \cos^3 x - 3 \cos x$ (b) (ii) $x = \frac{\pi}{3}, \frac{5\pi}{3}$
- (a) $y - 4 = -\frac{1}{6}x$ (b) $5 \sin(2x + 0.927)$
(c) $x = -2.03, -0.46, 1.11, 2.68$
- (a) $f^{-1}: x \mapsto \left(\frac{1-x}{2}\right)^{\frac{1}{3}}$ (c) $x = \frac{1}{2}$ (d) $x = 0, y = -1$

June 2008

- (a) $x = \frac{1}{2}(\ln 2 - 1)$ (b) $y = 16x + 16 - 8 \ln 2$
- (a) $R = 13, \alpha = 1.176$ (b) $x = 0.0849$
(c) (i) $R_{\max} = 13$ (ii) $x = 1.176$
- (c) $P(-1, 2), Q(0, 1), R(1, 0)$ (d) $x = -6$
- (b) $\left(0, \frac{1}{4}\right)$ (c) $\left(0, \frac{1}{4}\right)$ (d) $x = \pm\sqrt{5}$
- (b) $\theta = 11.5^\circ, 168.5^\circ$
- (a) (i) $e^{3x}(\sin x + 7 \cos x)$ (ii) $3x^2 \ln(5x + 2) + \frac{5x^3}{5x + 2}$
(c) $x = 1, -3$
- (c) $x_1 = 1.4371, x_2 = 1.4347, x_3 = 1.4355$

January 2009

- (a) $\frac{46}{3}$ (b) $\frac{2x^2 \cos 2x - 2x \sin 2x}{x^4}$
- (a) $\frac{1-x}{x-3}$
- $y = \frac{1}{2}x + \frac{\pi}{4}$
- (a) $g(x) \geq 1$ (c) $fg(x) \geq 3$ (d) $x = 0, 6$
- (a) (ii) $\theta = \frac{\pi}{18}, \frac{5\pi}{18}$
- (a) $(-1, -3e^{-1} - 1)$
(b) $x_1 = 0.2596, x_2 = 0.2571, x_3 = 0.2578$
- (a) $5 \cos\left(\theta - \frac{4}{3}\right)$ (b) max value = 5 where $\theta = \alpha = \frac{4}{3}$
(c) 5° (d) $t = 15.5$

June 2009

1. (a) $x_1 = 2.32$, $x_2 = 2.371581451\dots$, $x_3 = 2.355593575\dots$,
 $x_4 = 2.360436923\dots$
2. (b) 120°
3. (a) 80 (b) 12.6286... (c) $\frac{dP}{dt} = 16e^{\frac{t}{5}}$ (d) 250
4. (i) (a) $\frac{dy}{dx} = 2x \cos 3x - 3x^2 \sin 3x$
(b) $\frac{dy}{dx} = \frac{\left(\frac{2x}{x^2+1}\right)(x^2+1) - 2x \ln(x^2+1)}{(x^2+1)^2}$ (ii) $2x - 3y + 5 = 0$
5. (c) $f(x) > -k$ (d) $f^{-1}(x) = \frac{1}{2} \ln(x+k)$ (e) $x > -k$
6. (c) $5 \cos(2x - 36.87)$ (d) $x = 51.6^\circ, 165.2^\circ$
7. (c) $x = \ln 4$ or $x = 0$
8. (a) $\sin 2x = 2 \sin x \cos x$ (b) 0.13, 1.44

January 2010

1. $\frac{4}{3(x-1)(3x+1)}$
2. (b) $x_2 = 2.345$, $x_3 = 2.037$, $x_4 = 2.059$
3. (a) $\sqrt{34} \cos(x + 0.5404)$ (b) $x = (0.27, 4.93)$
4. (i) $\frac{2}{x^2+1} - \frac{1}{x^2 \ln(x^2+1)}$
7. (b) $2e^{2x} \sec 3x + 3e^{2x} \sec 3x \tan x$
(c) $a = -0.196$, $b = 0.812$
8. (a) $x = 22.5, 112.5$
9. (i) (a) $x = \frac{e^5 + 7}{3}$ (b) $x = \frac{-2 + \ln 15}{7 + \ln 3}$
(ii) (a) $\frac{1}{2} \ln(x-3)$ (b) $(x-1)^2 + 3$, $y > 3$

June 2010

1. (b) 26.6° , -153.4°
2. $x - 18y + 52 = 0$
3. (c) $x_1 = 1.3038$, $x_2 = 1.2867$, $x_3 = 1.2917$
4. (b) $x = -\frac{10}{3}$ (c) $fg(2) = 11$ (d) $-3 \leq g(x) \leq 6$
5. (a) (0, 2) (b) $x = \frac{1}{2}$ (c) $(4x-5)e^{-x} - (2x^2 - 5x + 2)e^{-x}$
(d) $(1, -e^{-1})$, $\left(\frac{7}{2}, 9e^{\frac{7}{2}}\right)$
6. (a) (i) (3, 4) (ii) (6, -8) (c) $f(x) = (x-3)^2 - 4$
7. (a) 0.6435 (b) (i) 2.5 (ii) 2.21 (c) 4.41 (d) 14:06, 18:43
8. (a) $\frac{2x-1}{x-3}$ (b) $x = \frac{3e-1}{e-2}$

January 2011

1. (a) $25 \cos(x + 1.287)$ (b) 25 (c) $x = 3.84, 6.16$
2. (a) $\frac{4x+1}{2x-1}$ (c) $f'(x) = \frac{-6}{(2x-1)^2}$, $f'(2) = -\frac{2}{3}$
3. $\theta = 54^\circ, 126^\circ, 198^\circ, 342^\circ$
4. (a) $A = 70$ (c) $2.426^\circ \text{C per minute}$
5. (a) $A(1, 0)$, $B(8, 0)$ (b) $f'(x) = -\ln x + \frac{8-x}{x}$
(e) $x_1 = 3.529$, $x_2 = 3.538$, $x_3 = 3.534$
6. (a) $f^{-1}(x) = \frac{3+5x}{x+2}$ (b) $-9 \leq g(x) \leq 4$ (c) -6
(d) 5 (f) $-9 \leq x \leq 4$
7. (b) $y = -2x + (\pi + 3)$
8. (b) $\frac{dx}{dy} = 2 \sec 2y \tan 2y$ (c) $\frac{dy}{dx} = \frac{1}{2x\sqrt{x^2-1}}$

June 2011

- (a) $\frac{2x+3}{(x^2+3x+5)}$ (b) $\frac{-x \sin x - 2 \cos x}{x^3}$
- (b) $x_1 = 0.80219$, $x_2 = 0.80133$, $x_3 = 0.80167$
- (a) $R = (0, -6)$ (b) $R = (-4, 3)$
- (a) $f^{-1}(x) = e^{4-x} - 2$ (b) $x \leq 4$ (c) $fg(x) = 4 - x^2$
(d) $fg(x) \leq 4$
- (a) $p = 7.5$ (c) $t = 4$
- (b) (ii) $x = 22.5^\circ$, 112.5° , 202.5° , 292.5°
- (b) $y + \frac{5}{2} = \frac{4}{15}(x+1)$
- (a) $\sqrt{13} \cos(3x + 0.983)$ (c) $x = 0.196$

January 2012

- (a) $2x \ln 3x + x$ (b) $\frac{4x \cos 4x - 3 \sin 4x}{x^4}$
- (a) $(-5, 0)$, $(0, -12)$ (b) $(-3, 0)$, $(2, 4)$
- (a) 20 mm^2 (b) 12.28 p.m.
- $\left(y - \frac{\pi}{4}\right) = -8(x - 2\sqrt{3})$
- 6.5° , 53.5° , 126.5° , 173.5°
- (c) $x_1 = 1.921$, $x_2 = 1.910$, $x_3 = 1.908$
- (b) $f^{-1}(x) = \frac{1+x}{2x}$ (c) $x > 0$ (d) $x = e^4 = 1$
- (c) $\frac{5\pi}{12}$, $\frac{11\pi}{12}$

June 2012

- $\frac{6}{(3x-2)(3x+1)}$
- (b) $x_1 = 0.41$, $x_2 = 1.20$, $x_3 = 1.31$
- (a) $\frac{2\pi}{9}$ (b) $y = -\frac{1}{3}x$
- (a) $(-1.5, 0)$ and $(0, 5)$ (b) $(0, 5)$ (c) $(0, 10)$ and $(-0.5, 0)$
- (a) $\frac{4}{(2 \sin \theta \cos \theta)^2} - \frac{1}{\sin^2 \theta}$ (c) $\theta = \frac{\pi}{3}, \frac{2\pi}{3}$
- (a) $f(x) > 2$ (b) $fg(x) = x + 2$ (c) $\ln 2 - \frac{3}{2}$
(d) $f^{-1}(x) = \ln(x-2)$, $x > 2$ (e) $(0, 3)$, $(3, 0)$
- (a) (i) $x^{-\frac{1}{2}}(\frac{1}{2} \ln 3x + 1)$ (ii) $\frac{80x}{(2x-1)^6}$ (b) $\frac{3}{18+2x^2}$
- (a) $R = 25$, $\alpha = 73.7^\circ$ (b) 113.1° , 173.1°
(c) $7 \cos 2x - 24 \sin 2x + 7$ (d) 32

January 2013

- (a) $w = \frac{1}{2}$ (b) $y = 160x - 112$
- (b) $x_1 = 2.3863$, $x_2 = 2.2847$, $x_3 = 2.3125$
- (a) $ff(-3) = 2$
- (a) $a = 0.927$ (b) (i) 2 (ii) $\theta = 4.07$
- (i) (a) $3x^2 \ln 2x + x^2$ (b) $3(x + \sin 2x)^2 \times (1 + 2 \cos 2x)$
- (i) $1 + \frac{\sqrt{2}}{2}$ or $1 + \frac{1}{\sqrt{2}}$ (ii) (b) 0° , 30° , 150° , 180°
- (b) $h'(x) = \frac{10-2x^2}{(x^2+5)^2}$ (c) $0 \leq h(x) \leq \frac{\sqrt{5}}{5}$
- (a) $\text{£}19\,500$ (b) $t = 4 \ln 2$
(c) Decrease = $\text{£}593$ per year

June 2013

1. $a = 3, b = -2, c = 7, d = -8, e = 24$
3. (b) $\theta = 7.8, 97.8, 187.8, 277.8$
4. (a) $(0, -16)$ and $(-1, 25e^{-2} - 16)$
 (c) $x_1 = 0.485, x_2 = 0.492, x_3 = 0.489$ (d) $\alpha = 0.49$
5. (a) $\frac{dx}{dy} = 6\sec^2 3y \tan 3y$ (c) $\frac{d^2x}{dy^2} = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$
6. (a) $x = \frac{7}{5}$ (b) $x = \frac{-1 + \ln 10}{3 + \ln 2}$
7. (a) $0 \leq f(x) \leq 10$ (b) $ff(0) = 3$ (c) $g^{-1}(x) = \frac{5x-4}{3+x}$ (d) $x = 0.4$
8. (a) $R = 25, \alpha = 73.74^\circ$ (b) $\frac{21}{25}$ (c) 7.29 m (d) $\theta = 133.7, 13.7$

June 2013 (R)

1. $\frac{1}{x+4}$
3. (a) $R = 5\sqrt{2}, \alpha = 8.1^\circ$ (b) $x = 53.1^\circ, 323.1^\circ$ (c) $k = \pm\sqrt{50}$
4. (a) $f(x) \geq 3$ (b) $fg(1) = 5$
 (c) $g^{-1}(x) = \frac{3-x}{4}$ (d) $x = 0, x = 0.5$
5. (a) $\frac{-2\sqrt{x} \sin 2x - \frac{1}{2}x^{-\frac{1}{2}} \cos 2x}{x}$ (b) $\mu = 6$ (c) $\frac{dy}{dx} = \frac{3}{\sqrt{4-x^2}}$
6. (i) $\lambda = \frac{1}{2}$ (ii) $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \pi$
7. (a) $x = -0.382, x = -2.618$
 (b) $f(x) = e^{x^2}(2x+3) + (x^2+3x+1)e^{x^2} \times 2x$
 (d) $x_1 = -2.420, x_2 = -2.427, x_3 = -2.430$
8. (a) 1000 (b) 8000 (c) $k = 0.386$
 (d) 6970 (e) 346